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**INEG 3313 - Probability and Statistics**

**Week 3 Homework (100 points)**

For each problem, you can solve by hand or use Excel to help. You need to show all work in either case.

**Question 1 (p90, 3-45).** The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.

1. What is the probability that a sample’s strength is less than 6250 kg/cm2?
2. What is the probability that a sample’s strength is between 5800 and 5900 kg/cm2?
3. What strength is exceeded by 95% of the samples?

**Answer:** (Provide equation/expression for each of the subproblem, and show your work or Excel function and the value for your final answer.)

a)

μ=6000 kg/cm²

σ=100 kg/cm²

X=6250 kg/cm2

Z=(X−μ​)/ σ

Z = (6250 kg/cm2 - 6000 kg/cm²) / 100 kg/cm²

Z = 2.5

Using Standard Normal Table or =NORM.DIST(6250,6000,100,TRUE)

P(X<6250) = 0.9938

b)

μ=6000 kg/cm²

σ=100 kg/cm²

X1=5800 kg/cm2

X2=5900 kg/cm2

Z = 5800 kg/cm2 - 6000 kg/cm²) / 100 kg/cm²

Z = 5900 kg/cm2 - 6000 kg/cm²) / 100 kg/cm²

Z1 = -2.0

Z2 = -1.0

Calculate Prob

P( 5800 < X < 5900 ) = P( Z < −1.0 )− P( Z < −2.0)

=NORM.DIST(5800,6000,100,TRUE)

P( Z < −2.0 ) = 0.02275

=NORM.DIST(5900,6000,100,TRUE)

P( Z < −1.0 ) = 0.1587

P( 5800 < X < 5900 ) = 0.136

c)

Using =NORM.S.INV(0.05) to get Z value

Z = -1.645

μ=6000 kg/cm²

σ=100 kg/cm²

Z=(X−μ​)/ σ

X=μ + Z ⋅ σ

X = 6000 + ( −1.645 ) ⋅ 100

X=5835.5 kg/cm²

**Question 2 (p118, 3-140**). The time to failure of a certain type of electrical component is assumed to follow an exponential distribution with a mean of 4 years. The manufacturer replaces free all components that fail while under warranty (guarantee in the textbook).

1. What percentage of the components will fail in 1 year?
2. What is the probability that a component will fail in 4 years? (skip original part b in textbook)
3. If the manufacturer wants to replace a maximum of 3% of the components, for how long should the manufacturer’s stated warranty on the component be?
4. By redesigning the component, the manufacturer could increase the life. What does the mean time to failure have to be so that the manufacturer can offer a 1-year warranty, yet still replace at most 3% of the components?

**Answer:** (Provide equation/expression for each of the subproblem, and show your work or Excel function and the value for your final answer.)

λ = 0.25

1. F(t)=1−e−λt  
   F(1) = 1 – e −λ⋅1 = 1 – e-0.25

=EXPONDIST(1, 0.25, TRUE)  
0.2212 or 22.12%

1. F(t)=1−e−λt  
   F(1) = 1 – e −λ⋅4 = 1 – e-1

=EXPONDIST(4, 0.25, TRUE)  
0.6321 or 63.21%

1. F(t)=1−e−λt =0.03

e−λt = 1 – 0.03   
e−λt = 0.97

−λt=ln(0.97)

t=ln(0.97)​/ −λ

t=ln(0.97)​/ −0.25

Excel Calculation

=(-LN(0.97))/0.25

T = 0.122 Years or 44.5 Days Approximately

1. F(1)=1−e−λ′⋅1=0.03

e−λ′⋅1=0.97

ln both sides

−λ′=ln(0.97)

λ′=-ln(0.97)

λ′=0.0305

Mean time to failure = 1/ λ′ = 1/0.0305 = 32.79 Years